Aufgabe 2. Minimierung

a)

	a	b	c	f(abc)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$f(abc) = (\bar{a}b\bar{c}) \vee (\bar{a}bc) \vee (a\bar{b}c) \vee (abc)$

<table>
<tr><td colspan="2">a</td><td colspan="2">$\bar{a}$</td><td></td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>b</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>0</td><td>$\bar{b}$</td></tr>
<tr><td>$\bar{c}$</td><td colspan="2">c</td><td>$\bar{c}$</td><td></td></tr>
</table>

$$f(abc) = (ac) \vee (c\bar{b}) \vee (\bar{a}b\bar{c})$$
$$f(abc) = \overline{\overline{((ac) \vee (c\bar{b}) \vee (\bar{a}b\bar{c}))}}$$
$$f(abc) = \overline{(\overline{(ac)} \wedge \overline{(c\bar{b})} \wedge \overline{(\bar{a}b\bar{c})})}$$
$$f(abc) = \overline{((\bar{a} \vee \bar{c}) \wedge (\bar{c} \vee b) \wedge (a \vee \bar{b} \vee c))}$$
$$f(abc) = \overline{(\bar{a} \vee \bar{c})} \vee \overline{(\bar{c} \vee b)} \vee \overline{(a \vee \bar{b} \vee c)}$$

b)

	a	b	c	d	f(abcd)
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0

8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	0

$f(abcd) = (\bar{a}\bar{b}\bar{c}\bar{d}) \vee (\bar{a}\bar{b}\bar{c}d) \vee (\bar{a}\bar{b}c\bar{d}) \vee (\bar{a}\bar{b}cd) \vee (\bar{a}bc\bar{d}) \vee (a\bar{b}c\bar{d}) \vee (abc\bar{d})$

	$\bar{a}$		a		
$\bar{b}$	1	1	1		$\bar{d}$
	1	1			d
b					
		1	1		$\bar{d}$
	$\bar{c}$	c		$\bar{c}$	

$$f(abcd) = (\bar{a}\bar{b}) \vee (c\bar{d})$$
$$f(abcd) = \overline{\overline{((\bar{a}\bar{b}) \vee (c\bar{d})o)}}$$
$$f(abcd) = \overline{(\overline{(\bar{a}\bar{b})} \wedge \overline{(c\bar{d})})}$$
$$f(abcd) = \overline{((a \vee b) \wedge (\bar{c} \vee d))}$$
$$f(abcd) = \overline{(a \vee b)} \vee \overline{(\bar{c} \vee d)}$$

c)

$$\oplus = XOR$$

$f(abcd) = ((a \oplus d) \vee c \wedge d) \wedge \neg b \vee b \wedge \neg(c \oplus d)$

	a	b	c	d	f(abcd)
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	

12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

$$f(abcd) = (\bar{a}\bar{b}\bar{c}\bar{d}) \vee (\bar{a}\bar{b}\bar{c}d) \vee (\bar{a}\bar{b}c\bar{d}) \vee (\bar{a}b\bar{c}\bar{d}) \vee (\bar{a}bc\bar{d}) \vee (a\bar{b}\bar{c}\bar{d}) \vee (a\bar{b}c\bar{d}) \vee (ab\bar{c}\bar{d}) \vee (abc\bar{d})$$

<table>
<tr><td></td><td colspan="2">$\bar{a}$</td><td colspan="2">a</td><td></td></tr>
<tr><td rowspan="2">$\bar{b}$</td><td>1</td><td>1</td><td>1</td><td>1</td><td>$\bar{d}$</td></tr>
<tr><td>1</td><td></td><td></td><td></td><td rowspan="2">d</td></tr>
<tr><td rowspan="2">b</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>$\bar{d}$</td></tr>
<tr><td></td><td>$\bar{c}$</td><td colspan="2">c</td><td>$\bar{c}$</td><td></td></tr>
</table>

$$f(abcd) = \bar{d} \vee (\bar{a}\bar{c}\bar{b})$$
$$f(abcd) = \bar{d} \vee \overline{(a \vee c \vee b)}$$

3. $f(abcd) = ((a \oplus d) \vee cd)\bar{b}b\overline{(c \oplus d)}$

	a	b	c	d	f(abcd)
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

	$\bar{a}$	a	

$\bar{b}$				1	$\bar{d}$
		1	1	1	d
b		1	1		
				1	$\bar{d}$
	$\bar{c}$	c		$\bar{c}$	

a) Implikanten 0. Ordnung:
$ab\bar{c}\bar{d}$
$a\bar{b}\bar{c}\bar{d}$
$a\bar{b}\bar{c}d$
$a\bar{b}cd$
$\bar{a}\bar{b}cd$
$\bar{a}bcd$
$abcd$

b) Primimplikanten nach KV Diagramm

cd
$a\bar{b}d$
$a\bar{b}\bar{c}$
$a\bar{c}\bar{d}$

c) Minimale Überdeckung für f(abcd)
cd
$a\bar{b}\bar{c}$
$a\bar{c}\bar{d}$

Disjunktive Normalform zu f(abcd)
$(\bar{a}\bar{b}cd) \vee (\bar{a}bcd) \vee (a\bar{b}\bar{c}\bar{d}) \vee (a\bar{b}cd) \vee (ab\bar{c}\bar{d}) \vee (abcd)$