1.a)

$n = \{a_1, a_2, a_3, a_4, a_5\}$

$$10111$$
$$01212$$
$$2 - 10 - 1 - 2$$
$$-12313$$
$$10111$$
$$1212$$
$$-1 - 2 - 3 - 4$$
$$2424$$
$$10111$$
$$1212$$
$$-2 - 2$$
$$0$$
$$\Rightarrow rg(n) = 3$$

b)

$$x_1 + x_3 + x_4 = 1$$
$$x_2 + 2x_3 + x_4 = 2$$
$$2x_1 - x_2 - x_4 = -2$$
$$-x_1 + 2x_2 + 3x_3 + x_4 = 3$$

$$-x_1 + 2x_2 + 3x_3 + x_4 = 3$$
$$2x_2 + 4x_3 + 2x_4 = 4$$
$$x_2 + 2x_3 + x_4 = 2$$
$$3x_2 + 6x_3 + x_4 = 4$$

$$-x_1 + 2x_2 + 3x_3 + x_4 = 3$$
$$2x_2 + 4x_3 + 2x_4 = 4$$
$$0 = 0$$
$$4x_4 = 4$$

$$-x_1 + 2x_2 + 3x_3 + x_4 = 3$$
$$2x_2 + 4x_3 + 2x_4 = 4$$
$$0 = 0$$
$$x_4 = 1$$

$x_1 = t, x_3 = t, x_2 = 1 - 2t x_4 = 1 Basis = \{a_1, a_2, a_3, a_4\}$

c)

$$\begin{pmatrix}1\\0\\2\\-1\end{pmatrix} = 1 * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix} + 0 * \begin{pmatrix}1\\2\\0\\3\end{pmatrix} + 0 * \begin{pmatrix}1\\1\\-1\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\\-1\\2\end{pmatrix} = -\frac{1}{2} * \begin{pmatrix}1\\2\\0\\3\end{pmatrix} + \frac{1}{2} * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix} + 0 * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\0\\3\end{pmatrix} = 0 * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix} + 1 * \begin{pmatrix}1\\2\\0\\3\end{pmatrix} + 0 * \begin{pmatrix}1\\1\\-1\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\1\\-1\\1\end{pmatrix} = 0 * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix} + 0 * \begin{pmatrix}1\\2\\0\\3\end{pmatrix} + 1 * \begin{pmatrix}1\\1\\-1\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\-2\\3\end{pmatrix} = -\frac{1}{2} * \begin{pmatrix}1\\0\\2\\-1\end{pmatrix} + \frac{1}{2} \begin{pmatrix}1\\2\\0\\3\end{pmatrix} + 1 * \begin{pmatrix}1\\1\\-1\\1\end{pmatrix}$$

2.

$$x_1 + 2x_2 = 0$$
$$x_1 - 4x_2 + 2x_3 = 0$$
$$x_1 + 11x_2 - 3x_3 = 0$$
$$x_1 + x_2 = 0$$

$$x_1 + 2x_2 = 0$$
$$+6x_2 - 2x_3 = 0$$
$$-9x_2 + 3x_3 = 0$$
$$0 = 0$$

$$x_1 + 2x_2 = 0$$
$$+6x_2 - 2x_3 = 0$$
$$0 = 0$$
$$0 = 0$$

$Gleichungssystemnichttriviallösbar \Rightarrow linearabhänig$

3.

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 3 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix} B = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & 1 \\ -3 & -2 & 0 \end{pmatrix}$$

$$1-111-21$$
$$20 \quad 4-1$$
$$-10 \quad -83$$

$$1-111-21$$
$$20 \quad 4-1$$
$$0 \quad 1$$

$$\Rightarrow rg(A) = 2 \Rightarrow rg(B) = 3$$

$rg(A) \neq rg(B) \Rightarrow AundBhabenverschiedeneRänge.$

4.

$$A = (a_{ij})_{mn} B = (b_{ij})_{np}$$
$$\lambda \in K \lambda * (AB) = (\lambda A) * B = A * (\lambda B)$$

$$(\lambda A) * B = (\lambda a_{ij})_{mn} * (b_{ij})_{np} = \left(\sum_{k=1}^{n} \lambda a_{ik} b_{kj}\right)_{mp}$$
$$= \left(\lambda \left(\sum_{k=1}^{n} a_{ik} b_{kj}\right)\right)_{mp} = \lambda \left(\sum_{k=1}^{n} a_{ik} b_{kj}\right)_{mp}$$
$$= \lambda(A * B)$$

$$\lambda \left(\sum_{k=1}^{n} a_{ik} b_{kj}\right)_{mp} = \left(\lambda \sum_{k=1}^{n} a_{ik} b_{kj}\right)_{mp} = \left(\sum_{k=1}^{n} a_{ik} \lambda b_{kj}\right)_{mp}$$
$$= (a_{ij})_{mn} * (\lambda b_{ij})_{np} = A * (\lambda B)$$